ANGULAR MOMENTUM & Kepler's 3rd Law

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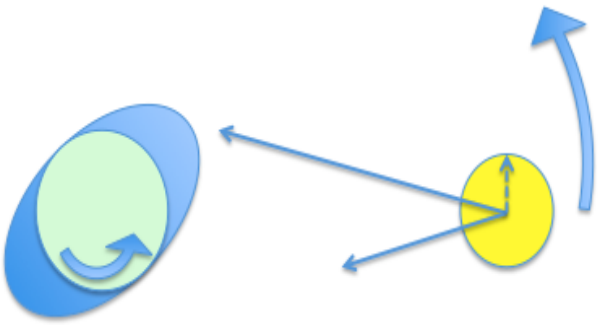
Scientists Link Moon's Tilt and Earth's Gold

The moon's orbit is askew, and two planetary scientists believe that they have come up with a good reason. Intriguingly, their idea also explains why gold and platinum are found in the Earth's crust, well within diggable reach. The moon is believed to have formed out of **a giant cataclysmic collision early in the history of the solar system when an interplanetary interloper the size of Mars slammed into Earth and lofted a ring of debris circling over the Equator. The debris coalesced into the moon.** At its birth, the moon was quite close to the Earth, probably within 20,000 miles. **Because of the tidal pulls between the Earth and moon, the moon's orbit has slowly been spiraling outward ever since, and as it does, Earth's pull diminishes, and the pull of the sun becomes more dominant.** By now, with the moon a quarter million miles from Earth, the sun's gravity should have tipped the moon's orbit to lie in the same plane as the orbits of the planets. But it has not. The moon's orbit is about 5 degrees askew. "That the lunar inclination is as small as it is gives us some confidence that the basic idea of lunar formation from an equatorial disk of debris orbiting the proto-Earth is a good one," said Kaveh Pahlevan, a planetary scientist at the Observatory of the Côte d'Azur in Nice, France. "But the story must have a twist." Writing in this week's issue of the journal Nature,



INTRODUCTION A: **Purpose of this application is to find orbital angular momentum ($I_{m1}\,\omega_{m1}$) of moon 600 million years ago. Then, find distance (R_{m1}) moon was from earth 600 million years ago. Since NO external torques exist on earth moon system their angular momentum is conserved. $L_{e1} + L_{m1} = L_{e2} + L_{m2}$ eq. 1**
Period of earth on axis 600 million years ago was 22 hrs./day. Earth period now is 23.93 hr./day. From MIT site the moment of inertia of earth (I_e) on it's axis is 9.72×10^{37} kg m^2.
QUESTIONS A: **(a) Find axis angular velocity of earth 600 M yrs. ago = $\omega_{e1} = 2\pi/T_{e1}$? (b) Find axis angular momentum of earth 600 M years ago $L_{e1} = I_e\,\omega_{e1}$? (c) Find axis angular velocity ($\omega_{e2} = 2\pi/T_{e2}$) of earth now? (d)Find earth axis angular momentum now = $I_e\omega_{e2} = L_{e2}$**

QUESTIONS A con.: **(e) Find orbital angular velocity of moon now = $\omega_{m2} = 2\pi/T_{m2}$? T_{m2} now = 27.3 days. Convert 27.3 days to seconds first. (f) Find current orbital moment of inertia of moon $I_{m2} = M_m R_{m2}^2$ =? $M_m = 7.35 \times 10^{22}$ kg. , $R_{m2} = 384.4 \times 10^6$ m, (g) Find Current moon angular momentum $L_{m2} = I_{m2}\,\omega_{m2}$? (h) Find angular momentum of moon L_{m1} 600 M years ago? Used eq. 1 : $L_{m1} = L_{e2} + L_{m2} - L_{e1}$**
(((ANSWERS A))) (a) $\omega_{e1} = 0.7933 \times 10^{-4}$ rad./s. (b) $L_{e1} = 7.71 \times 10^{33}$ kg. m^2/s., (c) $\omega_{e2} = 0.729 \times 10^{-4}$ rad./s. (d) $L_{e2} = 7.09 \times 10^{33}$ kg. m^2/s. , (e) $\omega_{m2} = 2.66 \times 10^{-6}$ rad./s. (f) $I_{m2} = 1.086 \times 10^6$ kg. m^2, (g) $L_{m2} = 28.89 \times 10^{33}$ kg.m^2/s. (NOTE: ast.cambidge.ac.UK site gives L_{m2} now as 29×10^{33} kg. m^2/s.), (h) $L_{m1} = 28.267 \times 10^{33}$ kg. m^2/s.
INTRODUCTION B: **$L_{m1} = (M_m R_{m1}^2)\,\omega_1 = (M_m R_{m1}^2)\,2\pi/T_{m1}$**
From Kepler's 3rd. law: $T_{m1} = R_{m1}^{3/2}[2\pi/(GM_e)^{1/2}]$
Thus, $L_{m1} = R_{m1}^{1/2}\,M_m\,(G\,M_e)^{1/2}$ eq.2
QUESTION B: **(a) Find distance moon was from earth 600 M years ago R_{m1} ? Use eq. 2 , G = 6.67×10^{-11} Nm^2/kg.^{2} , $M_e = 5.972 \times 10^{24}$ kg. (b) Find cm./yr. moon moves away from earth on average? NOTE: $R_{m2} = 3.844 \times 10^8$ m , t = 600×10^6 years**

ANSWERS B: **(a)$R_{m1} = 3.713 \times 10^8$ m , (b) ~ 2.2 cm./yr. Note: NASA says moon moves away at 3.8 cm./yr. Close!**